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STEP MODEL OF EROSION OF ELECTRODES. III. ADAPTATION TO ARBITRARY REGIMES OF ELECTROEROSION TREATMENT

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The step model of erosion of the cold electrodes of electric-arc heaters proposed in Inzh.-Fiz. Zh., **76**, No. 2, 36–42 (2003) *and* Ann. N.Y. Acad. Sci., **891**, 36–42 (1999) *has been modified for calculating the rate of electroerosion treatment of materials. The point heat source* [J. Appl. Phys., **66**, No. 9, 4095–4103 (1989)] *for determining the effective enthalpy of erosion has been reduced to a plane surface heat source equivalent in erosion and corresponding to the step model. The approximation analytical relations in dimensionless numbers have been obtained. The calculation results are in satisfactory agreement with the experimental data of different authors.*

Introduction. As the main physical processes governing the mechanism of electroerosion treatment different phenomena in gas discharges, conductivity and electric breakdown in liquids, underwater-explosion theory, etc. have been proposed (see [3–5] and the references therein). However, researchers were led to conclude that in determining the treatment rate, heat conduction and fusion of materials are of primary importance. In this work, we have modified the model of erosion of a cathode in electroerosion treatment of steels of different grades on the basis of experimental results [3, 6, 7]. The theoretical approach in this model is analogous to that successfully applied to calculation of the erosion of electrodes in electric-arc heaters and in certain special regimes of electroerosion treatment (see [1, 8–10]).

As has been shown in [10], even without modification of the step model is in good agreement with the experiments of 3D electroerosion machining to a template conducted in special (optimized) regimes of treatment [3]. In [3], the duration of a discharge pulse was selected so as to obtain the maximum rate of removal of the material. The analysis of these experimental data has shown (see [10]) that the effective enthalpy of erosion of the cathode is nearly the same for all the presented regimes. This corresponds to the concept of constancy of the effective enthalpy of erosion for a specific material, which has been adopted in the step model of erosion of the electrodes [1, 8, 9]. However, as a more comprehensive analysis of numerous experimental results demonstrates, this model (in the form presented in [10]) will not do for the regimes with arbitrarily selected electroerosion-treatment parameters for the following reasons:

(1) the step model [10] disregards the influence of a pause between pulses on erosion, although this influence is quite substantial in electroerosion treatment;

(2) the density of the current in the cathode spot is taken to be constant in the step model irrespective of the current and the discharge time.

Regarding the first of the above-mentioned reasons, we can note that discharge in electroerosion treatment has a tendency toward tying to the site of the previous pulse for a while, despite the fact that the discharge moves during the pulse itself, as has been shown in [11, 12]. Therefore, the local temperature under the discharge spot somewhat differs from the average temperature of the material and this difference depends on τ_{pa}, τ_{pu}, and I.

As far as the second reason is concerned, this assumption of the constancy of the current density fits the conditions in electric-arc heaters well. However, it makes it impossible to explain such a strong increase in the specific erosion with current (in kg·C^{-1}) that occurs in electroerosion treatment. Indeed, in a plane one-dimensional model with a constant heat-flux density $q_0 = jU$ (owing to the constancy of the current density j), the regime of cooling of the

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electrode under the spot after a pulse is identical for any current and depends only on the time of the pulse and the pause. In this case, the specific erosion (per 1 C) must be constant for any current if τ_{pu} and τ_{pa} are constant. At the same time, the experiments with observance of the conditions τ_{pu} = const and τ_{pa} = const but with a variable current (I = var) have revealed a strong influence of the discharge current on both the specific erosion (in kg·C^{-1}) and the rate of volume removal of the material. In electric-arc heaters, the temperature of the electrode increases with current since it is heated not only by the arc spot but also due to convection and radiation from the plasma flow, which leads to a strong increase in the erosion. In electroerosion treatment, these factors are absent but an increase in the erosion with current is observed.

The model of a point heat source [3] makes it possible to take account of changes in both the local temperature of the material and the current density (heat flux). In this model, heat is supplied to an infinitely small point which is identified with the cathode spot of electric discharge. Therefore, the isotherms in it are of spherical symmetry and the density of the current and the heat flux on the isotherm is a function of the current and the isotherm radius. The model makes it possible to calculate the radius R_f of the isotherm corresponding to the fusion temperature T_f according to the equation [3]

$$T_f = T_0 + \left(\frac{UI}{2\pi\lambda R_f}\right) \operatorname{erfc}\left(\frac{R_f}{2\sqrt{a\tau_{pu}}}\right). \qquad (1)$$

The product UI represents an integral heat flux supplied to the discharge spot. Equation (1) determines the motion of the fusion boundary with neglect of the heat of fusion.

As has been shown in [3], the value of the erosion measured experimentally differs from the volume of the molten metal calculated in accordance with the model of a point heat source; moreover, the relation between the volumes of the molten and removed metals depends on the current and the pulse duration. It is of interest and of importance to note: if we assume that the removed metal contains the quantity of heat expended on heating it to the fusion temperature (disregarding the heat of fusion), we obtain (see the table in [10]) a nearly constant effective enthalpy of erosion for all the regimes presented in [3]. Furthermore, the same effective enthalpy of erosion (nearly 4.5 MJ·kg^{-1}) has been determined in [10] with the use of the plane one-dimensional step model where the heat expended on erosion was calculated with change in the heat balance on the surface only owing to the change in the kind of boundary conditions on it after the beginning of fusion. This can indicate the proximity of the two approaches and the possibility of reducing one model to the other.

Unfortunately, the model of a point heat source makes it impossible to directly use the concept of the effective enthalpy of erosion, as has been done in the plane one-dimensional step model. The quantity q_{ef} was obtained there under the assumption of the stationary position of the fusion isotherm on the electrode surface in accordance with boundary conditions of the first kind [13]. Such an approach considerably simplifies the problem by replacing the heat of ablation by a change in the heat balance caused by the change in the kind of boundary conditions, as has been done in [1, 8, 9] and has totally proved itself in electric-arc heaters. Indeed, the problem of calculation of the actual motion of the fusion boundary (Stefan problem) is very difficult even for the case of a constant prescribed density of the heat flux [14]. Therefore, to obtain simple analytical and empirical relations for calculation of erosion in electroerosion treatment, in this work we will seek the possibility of combining the one-dimensional plane step model [1, 10] and the one-dimensional model of a point heat source [3].

Assumptions for the Modified Model. To take account of the variable character of the current density and the surface temperature in electroerosion treatment we make the following assumptions:

(1) the initial radius of the discharge plasma is very small, so that we can apply the model of a point heat source;

(2) the initial erosion occurring as the source radius approaches zero asymptotically can be disregarded;

(3) the thermal volt-equivalent of the discharge equal to the ratio of the energy arriving at the cathode to the current can be taken to be constant and independent of the current;

(4) to determine the radius of the fusion isotherm we can disregard the heat of fusion and take account only of the expenditure of heat on heating the material to the fusion temperature;

(5) for each radius R_f (calculated in accordance with the model of a point heat source) of the molten hemispherical volume formed by the end of the discharge we can find the equivalent radius r of a surface heat source to which the basic relations of the step model are applicable when the effective enthalpy of erosion of a given material of the cathode h_{ef} is constant irrespective of the regime parameters of electroerosion treatment.

Assumptions (1)–(4) have been substantiated (experimentally as well) in [3]. The new and most cardinal assumption (5) will be confirmed in this work.

Approach to the Construction of the Model. As has been shown in Part II (see [10]), the plane one-dimensional model with neglect of the residual heating from the previous pulses yields a constant value of the effective enthalpy of erosion h_{ef} for special regimes of electroerosion treatment (with an optimum duration of a pulse), which is expressed by the high correlation coefficient (R = 0.98) in linear approximation of the dimensional group $UW_sI\bar{\tau}\rho^{-1}$ as a function of the volume rate of removal of the material V. It follows that

$$V = \frac{UW_sI\bar{\tau}}{\rho h_{ef}} . \tag{2}$$

Expression (2), together with the quantities I, U, ρ, and h_{ef}, involves $\bar{\tau}$ determined as

$$\bar{\tau} = \frac{\tau_{pu}}{\tau_{pu} + \tau_{pa}} , \tag{3}$$

and

$$W_s = \frac{2}{\pi}\left(\arctan\sqrt{\frac{1-f_s}{f_s}} - \sqrt{f_s\,(1-f_s)}\right), \tag{4}$$

which is a function of

$$f_s = \frac{\pi}{4a\tau_{pu}}\left[\frac{(T_f - T)\,\lambda}{jU}\right]^2 . \tag{5}$$

To take account of the influence of τ_{pa} we must attach much importance to the influence of the residual local heating of the material under the spot from the previous pulse. The pulse time τ_{pu} and the current I determine the radius of the fusion boundary R_f and the temperature distribution before the pause in the vicinity of the cathode spot of the discharge. For distant points of the treated material the temperature T_0 = 300 K remains constant. On the other hand, the degree of relaxation and accordingly the surface temperature before the next discharge depend on τ_{pa}, τ_{pu}, and I.

Knowing the residual temperature T on the hemispherical surface of an eroded cavity before the next pulse, we can find such a compensating (equivalent) current density j_{eq} that would retain a constant value of the dimensionless fusion parameter f_s calculated according to Eq. (5) at an initial temperature of 300 K. Applying this approach to each experimental point of the special regimes in [10] (the regimes were taken from [3]), where satisfactory agreement was obtained between the step model and experiment under the assumption of the constancy of the initial surface temperature T_0 = 300 K, we can find a new variable current density j_{eq} for these experiments which depends on the residual temperature. The quantity j_{eq} will make it possible to find the radius of the equivalent surface heat source for each point in interpretation of the plane one-dimensional model:

$$r = \sqrt{I/(\pi j_{eq})} . \tag{6}$$

For the plane model we must take this variable radius r since it is a function of the pulse time, the current, and the residual temperature. Next, if the relation obtained from the "optimum" experiments [3] for j_{eq} has a more general character, it will also hold for other regimes of electroerosion treatment. This important assumption must be

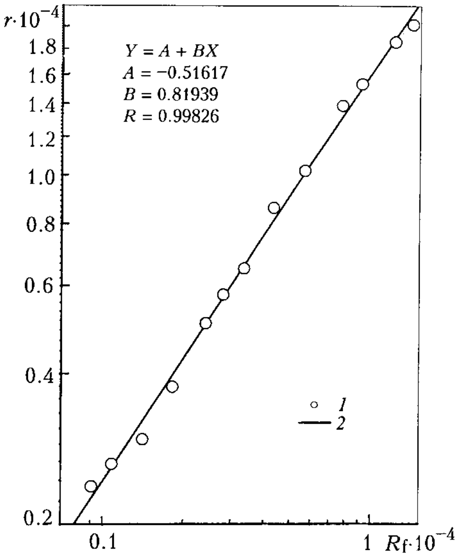

Fig. 1. Radius of the equivalent surface heat source r for the step model as a function of the radius of the fusion isotherm in the point heat source R_f: 1) calculations from Eqs. (5)–(7) for the experiments from [3] under the assumption that a constant value of the fusion parameter f_s is retained upon introduction of the residual temperature T into the calculation; 2) linear approximation for points 1.

checked on the materials of the experiments using the data on electroerosion treatment for other grades of steel and arbitrary regimes. For the calculations of the indicated experiments from [3] we took the thermophysical properties of steel according to [3] as well, namely: $\lambda = 56.1\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$, $C_p = 575\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}}$, $\rho = 7545\ \mathrm{kg \cdot m^{-3}}$, and $T_f = 1808$ K.

To determine j_{eq} we must calculate the residual temperature on the radius $R_f = f(\tau_{pu}, I)$ from the previous pulse. Since in actual practice the discharge moves during the pulse (according to [12]) and is not absolutely immobile, we will take account only of the heating from a single previous pulse, which will simplify the problem and will be checked thereafter by comparison with experiment. The residual temperature will be found by numerical solution of the differential equation (see [3])

$$\frac{1}{a}\frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial R^2} + \frac{2}{R}\frac{\partial T}{\partial R} \tag{7}$$

for the initial temperature distribution according to Eq. (1), where $T_0 = 300$ K. Having substituted the quantity $T(\tau_{pa})$ calculated according to Eq. (7) with the initial and boundary conditions determined from (1) into (5) for f_s for each experimental point, from the condition of conservation of the quantity f_s we find new, equivalent values of the current density f_{eq} for the plane heat source for each experimental point. The values of the equivalent radius r which are obtained from (6) and correspond to j_{eq} can be expressed by a simple power function of the radius of the fusion isotherm R_f, as is shown in Fig. 1:

$$r = 0.3 R_f^{0.82} \,. \tag{8}$$

Analytical relations are more convenient for practical calculations than the numerical solution of differential equations that are similar to (7). Therefore, we have approximated the numerical solutions obtained for Eq. (7) as functions of the dimensionless parameters Fo and Po. For our case

$$\mathrm{Fo} = \frac{a\tau_{pa}}{R_f^2} \tag{9}$$

TABLE 1. Regimes and Results of the Cathode Electroerosion Treatment of SVERKER3 and ARNE Steels [7] (V_1 and V_2 are the rates of removal of the material in two different operating regimes of the generator — isoenergetic and equifrequency ones [7])

I, A	τ_{pu}, μsec	τ_{pa}, μsec	V_1, $mm^3 \cdot min^{-1}$	V_2, $mm^3 \cdot min^{-1}$
SVERKER3 Steel				
4.5	10	3	4.4	4.5
4.5	50	7	4.2	4.2
9	10	3	13.8	15
9	50	7	17.4	18.3
9	200	15	10.4	10.4
13.5	20	2	35	37.6
13.5	100	9	33.5	34
18	10	2	41.2	42.2
18	20	2	51.6	52.7
18	50	4	57.1	56.5
18	100	9	57.4	57.2
18	200	15	53.7	53.6
27	20	2	80.6	80.7
27	100	9	102.5	102.5
27	500	32	86.4	85.1
ARNE Steel				
9	12	3	16.3	14.5
9	50	6	19.4	20.3
9	200	12	19.9	19.2
18	4	3	32.9	35.7
18	12	4	42.9	42
18	50	12	50.3	53.7
18	200	25	57.7	58.7
36	6	3	94	98.2
36	25	4	132.3	134.4
36	100	12	138.1	139.7

characterizes the time of cooling of the material during the pause, while

$$\mathrm{Po} = \frac{UI}{\lambda R_f T_f} \tag{10}$$

is the heat flux supplied to the material during the pulse [13]. To obtain the analytical relation in dimensionless numbers we have introduced the dimensionless residual temperature $\overline{T}$ determined as

$$\overline{T} = \frac{T - T_0}{T_f - T_0} . \tag{11}$$

Here T is the absolute value of the residual temperature.

As a result of the approximation of the numerical calculations in the range $\overline{T}$ = 0.025–0.9, Fo = $4.6 \cdot 10^{-4}$–23, and Po = 6.3–570, we have obtained the following empirical relation:

$$\overline{T} = \alpha\beta + \mathrm{erf}\,[(0.061 - 1.81\alpha)\,\beta] + \gamma\,, \tag{12}$$

where

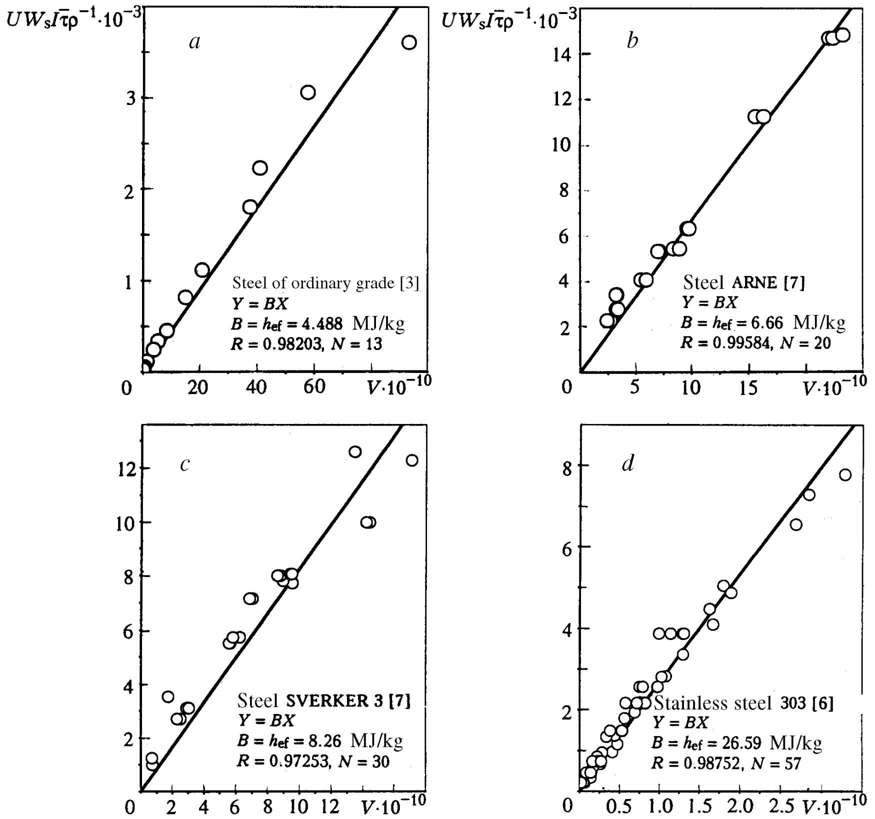

Fig. 2. Dimensional group $UW_sI\tau\rho^{-1}$ (J·m^{-3}·sec^{-1}·kg^{-1}) as a function of the rate of removal of the treated material V (m^3·sec^{-1}) for different grades of steel: a) ordinary grade from [3], b) ARNE from [7], c) SVERKER3 from [4], and d) 303 stainless steel from [6].

$$\alpha = 1.129\exp(2.107 - 1.076\,\mathrm{Po}) + 5.77\cdot10^{-3}\ln(\mathrm{Po}) - 1.66\cdot10^{-6}\,\mathrm{Po} - 0.115\,; \tag{13}$$

$$\beta = \ln\left(\frac{1}{4\,\mathrm{Fo}}\right) - 0.604\ln(\mathrm{Po}) + 3.607\exp(-\,0.1427\,\mathrm{Po}) + 0.187\,; \tag{14}$$

$$\gamma = 0.467 - 0.659\exp(-\,3.362 - 0.038\,\mathrm{Po}) + 8.625\cdot10^{-8}\,\mathrm{Po}^2\,. \tag{15}$$

The temperature $\bar{T}$ expressed in the variables Fo and Po is a function of the current I, the thermal volt-equivalent U, the pulse duration τ_{pu}, the pause duration τ_{pa}, and the thermophysical parameters of the cathode material (T_f, λ, and a).

Comparison with Experimental Data. To check the basic assumptions and the relations (1)–(5), (8), and (12) obtained we used the experimental data of different authors. The volume rate of removal of the material was calculated according to these relations for the wide range (presented in [3, 6, 7]) of operating regimes in electroerosion treatment of four different grades of steel with a wide range of variation of the thermophysical properties given below. The calculation results are presented in Figs. 2–4.

The data of [3], according to the description of DiBitonto et al., hold for the ordinary grade of steel. Norasetthekul et al. [6] give detailed results of other experiments on electroerosion treatment of 303 stainless steel according to the ASTM standard in a wide range of regimes. Among them are the volume rate of removal of the material V as a function of the current I for a constant duration of the pulse τ_{pu} and the pause τ_{pa} and as a function of the pulse duration τ_{pu} for I = const and τ_{pa} = const, and, furthermore, with constant regime parameters (I, τ_{pu}, τ_{pa}) but for different materials of the electrode (anode) tool. The thermophysical properties of 303 steel were as follows: λ = 14.6

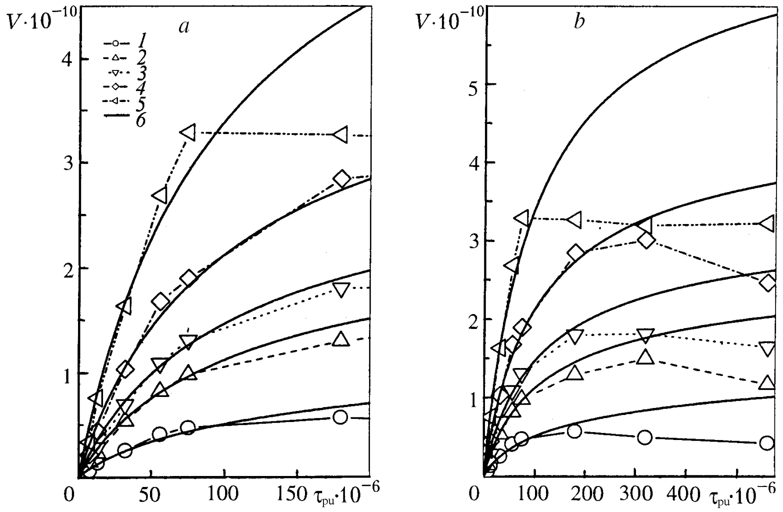

Fig. 3. Rate of removal of the treated material V (303 stainless steel [6]) ($m^3\cdot sec^{-1}$) vs. pulse duration τ_{pu} (μsec) and current density I (A) at a constant duration of the pause τ_{pa} = 100 μsec [a) range τ_{pu} = 0–200; b) 0–600 μsec]: 1) I = 8.3; 2) 14.8; 3) 18.3; 4) 24.8; 5) 37.1 A; 6) theoretical calculation for each value of the current according to Eqs. (1)–(5), (8), and (12).

$W\cdot m^{-1}\cdot K^{-1}$, $a = 3.728\cdot10^{-6}$ $m^2\cdot sec^{-1}$, ρ = 7800 $kg\cdot m^{-3}$ (according to the data supplied by the manufacturer [15]), and T_f = 1672 K (according to [16]).

The results of the electroerosion treatment of two grades of steel — ARNE and SVERKER3 — with an arbitrary variation of the operating parameters in a wide range are given in [7]. The data from [7] are presented in Table 1. The thermophysical properties of these steels were also taken according to the data supplied by the manufacturer (see [17]): λ = 32 $W\cdot m^{-1}\cdot K^{-1}$, $a = 8.92\cdot10^{-6}$ $m^2\cdot sec^{-1}$, ρ = 7800 $kg\cdot m^{-3}$, and T_f = 1808 K for ARNE and λ = 20.5 $W\cdot m^{-1}\cdot K^{-1}$, $a = 5.788\cdot10^{-6}$ $m^2\cdot sec^{-1}$, ρ = 7700 $kg\cdot m^{-3}$, and T_f = 1808 K for SVERKER3.

Figure 2 shows the dependence of the dimensional group $UW_s\bar{I}\tau\rho^{-1}$ calculated according to Eqs. (1)–(5), (8), and (12) on the volume rate of removal of the material V measured experimentally for each of the above steel grades. For the calculations we took U = 4.575 V according to [3]. We calculated the current density in the equivalent plane source j_{eq} for f_s (see Eq. (5)) from formula (6) with the use of the equivalent radius of the discharge spot $\underline{r}$ calculated in accordance with (1) and (8). The dimensionless residual temperature $\bar{T}$ was calculated according to (11) and (12). From Fig. 2 it is obvious that the dimensional group $UW_s\bar{I}\tau\rho^{-1}$ and the volume rate of removal of the material V for these steels are linearly dependent with a high correlation coefficient R (0.97 to 0.99). This result confirms the correctness of the model of ablation of a material with a constant effective enthalpy h_{ef} dependent just on the thermophysical properties of the material and independent of the regime parameters of electroerosion treatment.

From Fig. 2 we can determine h_{ef} according to relation (2) for each of the investigated materials. In this manner we have obtained h_{ef} = 4.49 $MJ\cdot kg^{-1}$ for ordinary steel [3], h_{ef} = 6.66 $MJ\cdot kg^{-1}$ for ARNE steel [7], h_{ef} = 8.26 $MJ\cdot kg^{-1}$ for SVERKER3 steel [7], and h_{ef} = 26.6 $MJ\cdot kg^{-1}$ for 303 stainless steel.

According to [6], the typical range of pulse duration used in electroerosion treatment in industry is 10 to 180 μsec. Therefore, Fig. 2 shows results of the processing of the experiments only in this range (for a pulse duration of up to 200 μsec) as the most important for industry. The influence of the pulse duration on the rate of removal of the material V with the other parameters (τ_{pa} and I) being constant is shown in Fig. 3.

Figure 3a shows the dependence of the rate of erosion of the cathode on the duration of a pulse τ_{pu} (according to [6]) in the range up to 200 μsec as well; the current I is taken as the parameter for each curve. Figure 3b shows the same experiments in the range of pulse duration up to 600 μsec. Furthermore, these figures show the theoretical curves calculated from the modified step model for each value of the current. The value of the effective enthalpy has been taken according to Fig. 2d. Good agreement between the calculated curves and experiment is seen in the range of pulse duration up to $\tau_{pu} < 200$ μsec; the measured values of the erosion are appreciably lower than the calculated values when the pulse duration is longer. According to [6], this is attributable to the inefficient removal of

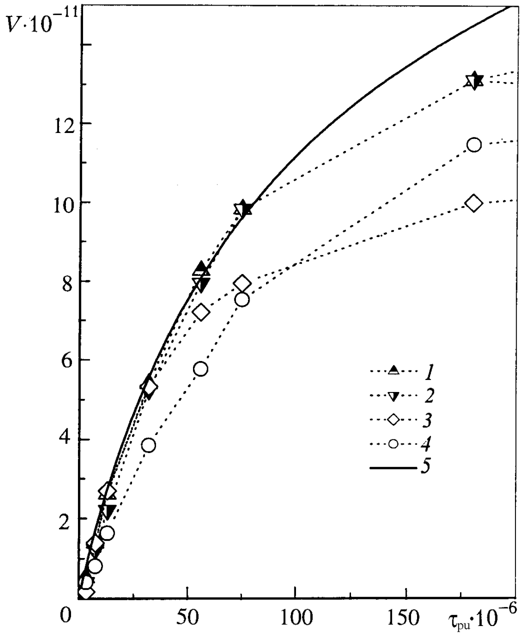

Fig. 4. Comparison of the rate of treatment of 303 stainless steel [6] in the case of employment of the anodes manufactured from different materials for a discharge current of 14.8 A and a pause duration of 100 μsec: 1, 2) two different samples of a special composite material from a zirconium-diboride matrix with a copper filler which differ in manufacturing methods; 3) graphite; 4) copper; 5) calculation according to Eqs. (1)–(5), (8), and (12).

the erosion products from the interelectrode gap and their deposition on the cathode again when the pulse duration is long.

The volume rate of removal of the material V as a function of the duration of the pulse τ_{pu} in the case of the anodes manufactured from different materials is shown in Fig. 4. We can see that the best agreement between theory and experiment is observed for short pulses. As τ_{pu} increases, the stratification of the experimental points and the distance between them and the calculated curve increase. For more resistant anodes the rate of removal of the cathode material is higher while the difference between the experimental and calculated data is smaller. This allows the assumption that the influence of the anode material manifests itself primarily as the effect of contamination of the interelectrode gap which is lower for more resistant anodes. Furthermore, the thermophysical properties of both the material of the anode and the dielectric can also exert an influence on the quantity U but it is, apparently, less substantial.

CONCLUSIONS

A modified step model of erosion of cold cathodes in nonstationary spots of electric discharges has been proposed and adapted to the conditions of electroerosion treatment. The model is based on the unification of the basic propositions of two different one-dimensional thermal models of erosion of cold cathodes — the plane one-dimensional step model [1, 2, 10] and the model of a point heat source [3]. This enabled us to use, for both electroerosion treatment and electric-arc heaters, the concept of the effective erosion enthalpy obtained through replacement of the actual physical processes on the moving boundary of two phases by a simple change of the kind of the boundary conditions on a virtual stationary surface. For practical application we have approximated the basic relations with the use of the dimensionless numbers $\text{Fo} = a\tau_{pa}/R_f^2$ and $\text{Po} = UI/(\lambda T_f R_f)$, taking account of the basic parameters of the process of electroerosion treatment: the current, the duration of a pulse and a pause, the thermal volt-equivalent of the discharge, and the thermophysical properties of the treated material.

The calculations according to the proposed modified step model of erosion have shown a high correlation ($R = 0.97$–0.99) with the experimental data obtained by different authors in electroerosion treatment of different grades of steel. This result confirms the basic concept of the model: the validity of the assumption that the effective enthalpy of ablation of each material as the basic characteristic of its erosion resistance in the process of electroerosion treatment is constant. The effective enthalpies of ablation for different grades of steel have been obtained in the range from 4.5 (ordinary grade of steel) to 26.2 $\mathrm{MJ \cdot kg^{-1}}$ (303 high-alloy austenitic stainless steel according to the ASTM standard).

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NOTATION

T_0, T_f, and T, initial temperature, fusion temperature, and residual temperature respectively, K; q_0, density of the heat flux supplied from the arc plasma to the spot, $\mathrm{W \cdot m^{-2}}$; U, volt-equivalent of the heat flux in the arc spot, V; I, current, A; λ, a, and C_p, thermal conductivity, thermal diffusivity, and specific heat, $\mathrm{J \cdot sec^{-1} \cdot m^{-1} \cdot K^{-1}}$, $\mathrm{m^2 \cdot sec^{-1}}$, and $\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$ respectively; R_f and r, radius of the fusion isotherm and radius of the equivalent plane heat source respectively, m; τ, time, sec; τ_{pu} and τ_{pa}, time of the pulse and the pause respectively, sec; $\bar{\tau}$, relative time of the pulse; V, volume rate of removal of the material, $\mathrm{m^3 \cdot sec^{-1}}$; W_s, dimensionless erosion energy for the step model; f_s, dimensionless parameter of fusion for the step model; ρ, density, $\mathrm{kg \cdot m^{-3}}$; h_{ef}, effective enthalpy of ablation of the material, $\mathrm{J \cdot kg^{-1}}$; j and j_{eq}, effective current density and current density in the equivalent plane source respectively, $\mathrm{A \cdot m^{-2}}$; Fo, Fourier number; Po, Pomerantsev number. Subscripts: 0, characteristic value of the quantity (for example, initial temperature T_0, time of heating to the fusion temperature τ_0); ef, effective value; f, fusion; pu, pulse; pa, pause; s, parameter for the step motion of the spot; eq, equivalent.

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